Suspension Modelling in the Lattice Boltzmann Method

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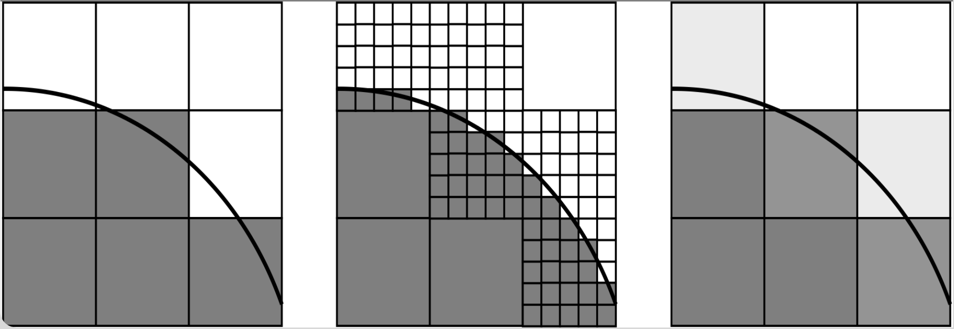
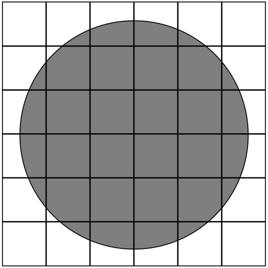
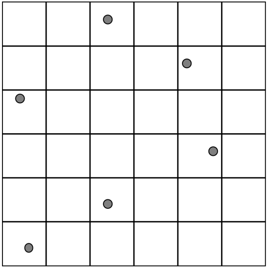


Structure

- Introduction
- Non resolved approach
    - Coupling approaches
- Resolved approach
    - Volume approximation techniques
    - Partially saturated method (PSM)
    - Homogeneous lattice Boltzmann method (HLBM)
    - Benchmark comparison
- Conclusion

Resolved and Non-resolved Approaches





Resolved approach

- Particle diameters d_p are greater than the cell length Δx
- Particles are simulated by different approaches that recover the shape

Non resolved approach

- Particle diameters d_p are smaller than the cell length Δx
- Particles are modelled by adding forces

Governing Equations

- Forced lattice Boltzmann equation

$$f_i(\boldsymbol{x} + \boldsymbol{c}_i \Delta t, t + \Delta t) = f_i(\boldsymbol{x}, t) + \Omega_i(\boldsymbol{x}, t) + \Delta t F_i(\boldsymbol{x}, t)$$

- Newton's law of motion

$$m_s \frac{d\boldsymbol{u}_s}{dt} = m_s \frac{d^2 \boldsymbol{x}_s}{dt^2} = F(\boldsymbol{x}, t)$$

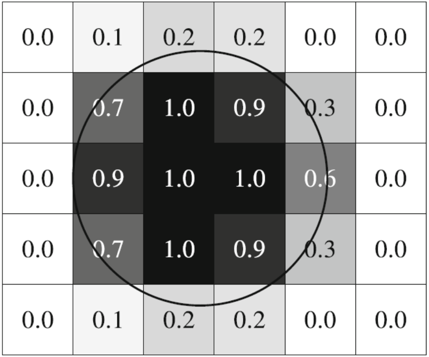
Coupling Approaches

- Volume fraction of solid phase:
  - **Dilute**: Low volume fraction, Impact of particles on the fluid flow can be neglected
  - **Dense**: High volume fraction, the fluid flow is affected by the presence of the particle
- One-way-coupling
  - Hydrodynamic forces acting on particles
- Two-way-coupling
  - + Particles forces acting on the fluid
- Four-way-coupling
  - + Particle-particle interaction (collision model)
  - + Particle wall interaction

Resolved Approach

- Solid volume fraction $\varepsilon(\boldsymbol{x},t)$

$$\varepsilon(\boldsymbol{x},t) = \frac{V_s(\boldsymbol{x},t)}{V_{total}(\boldsymbol{x},t)}$$



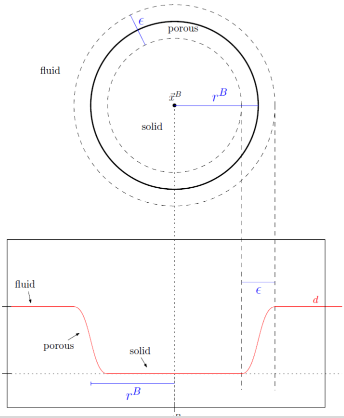
Volume Approximation Techniques

Karlsruher Institut für Technologie

- Estimation of the solid volume fraction $\varepsilon(\boldsymbol{x}, t)$ is difficult
- Compromise of computing time and accuracy
- Established methods:
  - Exact closed form solution
  - Polyhedral approximation
  - **Smoothed profile method**
  - **Supersampling**

Smoothed profile method

$$\varepsilon(\boldsymbol{x},t) = \begin{cases} 0 & \|\boldsymbol{x}-\boldsymbol{x}^B(t)\|_2 - r^B + \epsilon/2 \geq \epsilon \\ \sin^2\left(\frac{\pi(\|\boldsymbol{x}-\boldsymbol{x}^B(t)\|_2 - r^B + \epsilon/2)}{2\epsilon}\right) & \|\boldsymbol{x}-\boldsymbol{x}^B(t)\|_2 - r^B + \epsilon/2 \in (0,\epsilon) \\ 1 & \|\boldsymbol{x}-\boldsymbol{x}^B(t)\|_2 - r^B + \epsilon/2 \leq 0 \end{cases}$$

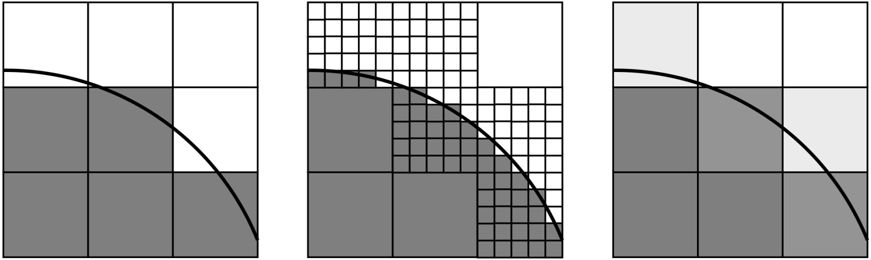


Supersampling

- Every cell, where $\varepsilon(\boldsymbol{x}, t) \in (0, 1)$, is divided in a subgrid
- Subgrid is constructed with a certain refinement level r
- Binary information of the subgrid is used to estimate $\varepsilon(\boldsymbol{x}, t)$

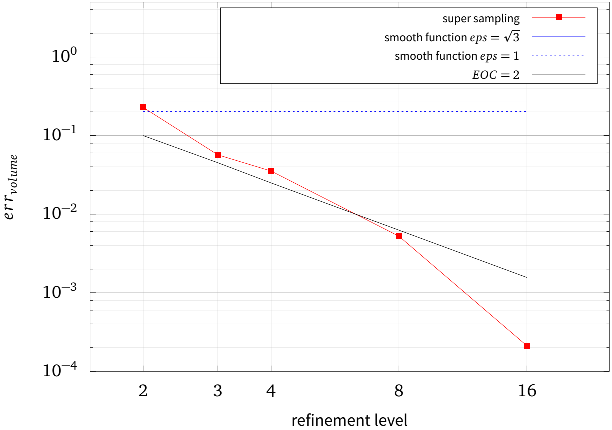
$$\varepsilon(\boldsymbol{x}, t) = \sum_{i=1}^{r^3} V_{s,i}^{subgrid}(\boldsymbol{x}, t)$$

$$V_{s,i}^{subgrid}(\boldsymbol{x}, t) = \begin{cases} 0 & false \\ \frac{1}{r^3} & true \end{cases}$$



Supersampling vs. Smoothed Profile

$$err_{volume} = \frac{1 - \frac{V_r}{V_{analytical}}}{1 - \frac{V_{staircase}}{V_{analytical}}}$$



PSM – Algorithm

- Collide and stream

$$f_i(\boldsymbol{x} + \boldsymbol{c}_i \Delta t, t + \Delta t) = f_i(\boldsymbol{x}, t) + B\Omega_i^s(\boldsymbol{x}, t) + (1 - B)\Omega_i^f(\boldsymbol{x}, t)$$

- Weighting factor $B(\boldsymbol{x}, t)$

$$B(\boldsymbol{x}, t) = \frac{\varepsilon(\boldsymbol{x}, t)\left(\frac{\tau}{\Delta t} - \frac{1}{2}\right)}{(1 - \varepsilon(\boldsymbol{x}, t)) + \left(\frac{\tau}{\Delta t} - \frac{1}{2}\right)}$$

- Solid collision operator $\Omega_i^s(\boldsymbol{x}, t)$

$$\Omega_i^s(\boldsymbol{x}, t) = \left[f_{\bar{i}}(\boldsymbol{x}, t) - f_{\bar{i}}^{\mathrm{eq}}(\rho, \boldsymbol{u}_s)\right] - \left[f_i(\boldsymbol{x}, t) - f_i^{\mathrm{eq}}(\rho, \boldsymbol{u}_s)\right]$$

- Fluid collision operator $\Omega_i^f(\boldsymbol{x}, t)$

$$\Omega_i^s(\boldsymbol{x}, t) = \frac{\Delta t}{\tau}\left(f_i^{\mathrm{eq}}(\rho_f, \boldsymbol{U}) - f_i(\boldsymbol{x}, t)\right)$$

PSM – Force and Torque

- Total hydrodynamic force $\boldsymbol{F}(t)$

$$\boldsymbol{F}(t) = \frac{(\Delta x)^3}{\Delta t} \sum_{\boldsymbol{x}_s} \left[B(\boldsymbol{x}_s, t) \sum_i \left(\Omega_i^s(\boldsymbol{x}_s, t) \boldsymbol{c}_{\bar{i}} \right) \right]$$

- Boundary torque $\boldsymbol{T}(t)$

$$\boldsymbol{T}(t) = \frac{(\Delta x)^3}{\Delta t} \sum_{\boldsymbol{x}_s} \left[B(\boldsymbol{x}_s, t)(\boldsymbol{x}_s - \boldsymbol{R}) \times \sum_i \left(\Omega_i^s(\boldsymbol{x}_s, t) \boldsymbol{c}_{\bar{i}} \right) \right]$$

HLBM – Algorithm

- Collide and stream

$$f_i(\boldsymbol{x} + \boldsymbol{c}_i \Delta t, t + \Delta t) = f_i(\boldsymbol{x}, t) + \Omega_i^f(\boldsymbol{x}, t)$$

- Weighting factor $B(\boldsymbol{x}, t)$

$$B(\boldsymbol{x}, t) = \varepsilon(\boldsymbol{x}, t)$$

- Fluid collision operator $\Omega_i^f(\boldsymbol{x}, t)$

$$\Omega_i^f(\boldsymbol{x}, t) = \frac{\Delta t}{\tau}\left(f_i^{\mathrm{eq}}(\rho_f, \bar{\boldsymbol{u}}) - f_i(\boldsymbol{x}, t)\right)$$

- Equilibrium velocity $\bar{\boldsymbol{u}}$

$$\bar{\boldsymbol{u}} = \boldsymbol{U} + B(\boldsymbol{x}, t)(\boldsymbol{u}_s - \boldsymbol{U})$$

HLBM – Force and Torque

- Momentum Exchange $g_i(\boldsymbol{x},t)$

$$g_i(\boldsymbol{x},t) = \mathbf{c}_i f_i(\boldsymbol{x},t) + \mathbf{c}_{\bar{i}} f_i(\boldsymbol{x} + \mathbf{c}_{\bar{i}}\Delta t, t)$$

- Total hydrodynamic force $\boldsymbol{F}(t)$

$$\boldsymbol{F}(t) = \frac{(\Delta x)^3}{\Delta t} \sum_{\boldsymbol{x}_s} \left[B(\boldsymbol{x}_s,t) \sum_i g_i(\boldsymbol{x},t) \right]$$

- Boundary torque $\boldsymbol{T}(t)$

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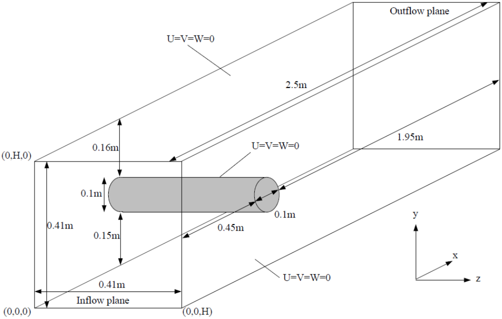
Schäfer Turek – Flow around a Cylinder

- Drag and lift coefficients

$$c_D = \frac{2F_w}{\rho \bar{u}^2 DH}, \qquad c_L = \frac{2F_a}{\rho \bar{u}^2 DH}$$



Schäfer Turek – Flow around a Cylinder

C_D	$N = 10$	$N = 20$	$N = 40$
HLBM $\epsilon = 2$	9.83	7.57	6.81
HLBM $\epsilon = 1$	8.15	6.90	6.51
HLBM $\epsilon = 0$	6.90	6.38	6.22
PSM	6.47	6.30	6.24
Bouzidi	6.35	6.22	6.19
Schäfer et al.	6.05-6.25		

Schäfer Turek – Flow around a Cylinder

$C_L(\times 10^{-2})$	$N=10$	$N=20$	$N=40$
HLBM $\epsilon = 2$	0.00	1.04	1.11
HLBM $\epsilon = 1$	0.00	0.66	0.87
HLBM $\epsilon = 0$	0.00	0.53	0.73
PSM	0.47	0.77	0.84
Bouzidi	13.20	2.50	1.25
Schäfer et al.		0.80-1.00	

Conclusion and Outlook

- Lattice Boltzmann provides resolved and non resolved approaches to simulate suspensions
- Efficient and accurate volume fraction approximation techniques are challenging
- Various resolved approaches offers a wide range of applications

Thank you for your attention!