PIRE ARM 2018: Turbulent Thermal Lattice Boltzmann Methods

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Outline

- Mathematical Model
- Thermal Lattice Boltzmann Method
- Validation
  - Validation Against Literature
- Summary

Thermal Flows – Mathematical Model

- Navier-Stokes equation for velocity and pressure distribution

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\frac{1}{\rho} \nabla p + \nu \Delta u + F$$

- Advection-diffusion equation for temperature distribution

$$\frac{\partial T}{\partial t} = -u \cdot \nabla T + \alpha \Delta T$$

- Coupling via Boussinesq approximation
  - Only the fluid density is a function of the temperature

$$\rho(T) \approx \rho_0 (1 - \beta (T - T_0))$$

  - Buoyancy force G:

$$G = \rho_0 \beta (T - T_0) g$$

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LBM for Velocity and Pressure

- Particle distribution function f_i:

$$f_i(x + c_i \Delta t, t + \Delta t) = f_i(x,t) - \frac{\Delta t}{\tau}\left(f_i(x,t) - f_i^{eq}(x,t)\right) + F_i$$

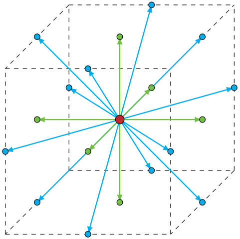
- With equilibrium distribution function f_i^{eq}:

$$f_i^{eq} = \omega_i \rho \left(1 + \frac{u \cdot c_i}{c_s^2} + \frac{(u \cdot c_i)^2}{2c_s^4} - \frac{u \cdot u}{2c_s^2}\right)$$

- For small velocities u, so that $Ma = \frac{|u|}{c_s} \ll 1$
- With relaxation time $\tau = \frac{1}{c_s^2}(\nu + \frac{1}{2}\Delta t)$
- Density ρ and velocity u:

$$\rho(x,t) = \sum_i f_i(x,t) \qquad u(x,t) = \frac{1}{\rho}\sum_i c_i f_i(x,t)$$

D3Q19



LBM for Temperature

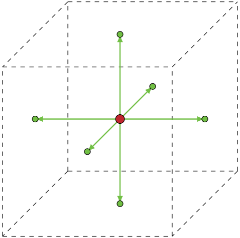
- 2nd particle distribution function g_i:

$$g_i(x + c_i\Delta t, t + \Delta t) - g_i(x,t) = -\frac{\Delta t}{\tau'}\left(g_i(x,t) - g_i^{eq}(x,t)\right)$$

- With equilibrium particle distribution g_i^{eq}:

$$g_i^{eq} = \omega_i T \left(1 + \frac{c_i \cdot u}{c_s^2}\right)$$

D3Q7



- With relaxation time $\tau' = \frac{1}{c_s^2}(\alpha + \frac{1}{2}\Delta t)$
- Temperature T:

$$T = \sum_i g_i$$

Thermal Large Eddy Simulation (LES)

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- Effective viscosity as a sum: molecular viscosity + turbulent viscosity

$$\nu_{\text{eff}} = \nu_0 + \nu_t = \nu_0 + (C_S \Delta)^2 \sqrt{2 \sum_{\alpha,\beta} S_{\alpha\beta} S_{\alpha\beta}} \tag{1}$$

- in LBM: strain rate $S_{\alpha\beta}$ is calculated locally from the cell's non-equilibirum

$$S_{\alpha\beta} = -\frac{1}{2\rho \tau_{\text{eff}} c_s^2} \Pi_{\alpha,\beta}^{(neq)} = -\frac{1}{2\rho \tau_{\text{eff}} c_s^2} \sum_q e_{i,\alpha} e_{i,\beta} (f_i - f_i^{(eq)}) \tag{2}$$

- Equivalent calculation for the effective temperature conductivity

$$\alpha_{\text{eff}} = \alpha_0 + \alpha_t = \alpha_0 + \frac{\nu_t}{Pr_t} \tag{3}$$

- With constant turbulent Prandtl number

$$Pr_t = \frac{\nu_t}{\alpha_t} = 0.86 \tag{4}$$

LBM for Thermal Flows

Double Distribution Function Algorithm

- 1st particle distribution function f_i
  - Collision step
  - Streaming step
- 2-way coupling via buoyancy force with Boussinesq approximation
  - Communicate the updated velocities from f_i to g_i
  - Update the force term of f_i with new temperatures from g_i
  - Calculate ν_{eff}, α_{eff} and corresponding τ and τ'
- 2nd particle distribution function g_i
  - Collision step
  - Streaming step
- Next time step

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Natural Convection in a Square Cavity

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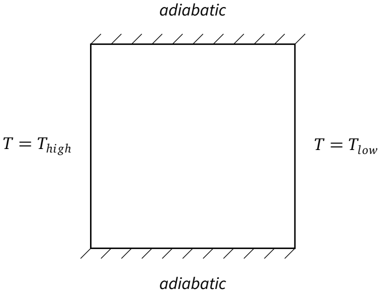
Benchmark according to Davis (1983):

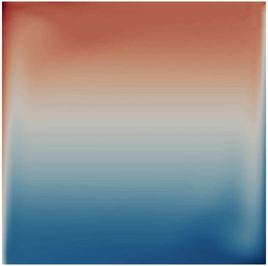
- $u_x = u_y = 0$ on all walls
- $T = T_{high}$ left, $T = T_{low}$ right
- Adiabatic on top and bottom
- $Pr = 0.71$ (air)
- $Ra = \frac{g\beta}{\nu a}(T_{high} - T_{low})L^3$
  $= 10^7; 10^8; 10^9; 10^{10}$



- Comparison:
  - $max(u_x)$ and its position
  - $max(u_y)$ and its position
  - Nu_0 at the left wall
  - Streamlines and temperature contours

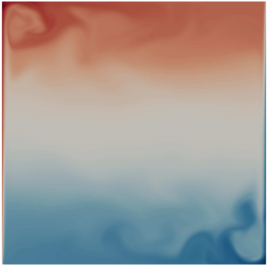
Thermal Cavity – Temperature Distribution

Ra = 10^7



Ra = 10^8

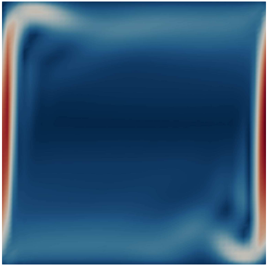
Ra = 10^9



Ra = 10^{10}

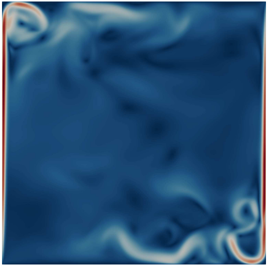
Thermal Cavity – Velocity Distribution

Ra = 10^7

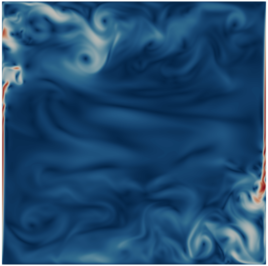


Ra = 10^8

Ra = 10^9



Ra = 10^{10}



Thermal Cavity – Literature Comparison

$$y^+ = \frac{\Delta x u_\tau}{\nu} = 2$$

Ra Grid used		10^7 (47x47x3)	10^8 (139x139x3)	10^9 (392x392x3)	10^{10} (1138x1138x3)
y_{max}	Dixit (2006)	0.851	0.937	0.966	0.9402
	Present	0.905	0.945	0.950	0.907
$u_{x,max}$	Dixit (2006)	164.236	389.877	503.24	2323
	Present	130.221	280.674	2277.140	2967.5
x_{max}	Dixit (2006)	0.020	0.0112	0.0064	0.4907
	Present	0.042	0.014	0.008	0.004
$u_{y,max}$	Dixit (2006)	701.922	2241.374	6820.07	21463
	Present	467.976	1860.270	6124.420	20533
$\overline{\mathrm{Nu}}_0$	Dixit (2006)	16.79	30.506	57.350	103.663
	Present	16.987	32.076	52.260	102.034

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Summary and Outlook

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Summary:

- 3D Lattice Boltzmann method (LBM) for thermal flows
- Addition for turbulent flows with Smagorinsky-LES-Model
- Good agreement with turbulent benchmark

Outlook:

- Resolved phase change simulation through coupling the heat equation with suspension modeling